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Development of Volume Equation and Taper Model for Young Teak Plantation: A Case Study from Teak Provenance Trial in Moeswe Area of Nay Pyi Taw

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Abstract

This study was conducted to develop double entry volume equation and taper model from the felled trees of young teak plantation in Moeswe area of Nay Pyi Taw. Three double entry volume equations were tested. For the homogeneity of variance, two equations were analyzed by weighted least square technique. For the same reason, one non-linear equation was transformed to logarithmic form which was linear. All the equations were evaluated by using Furnival's Index (FI). The equation with the lowest FI was selected as the best one for volume estimation for small dimensions of young teak trees. The range of diameter at breast height (D) was between 7 and 25 centimeters (cm) and total height (H) was ranged from 8 to 23 meters (m). The stump volume was excluded, and the volume can be predicted up to approximate 5 cm diameter. The second part of the study is to develop taper model for young teak trees with small dimensions in term of D and H. Three taper models, namely Bruce et.al. (1968) polynomial taper model, Kozak et.al. (1969) parabolic taper model and Demaerschalk-I (1972) taper model, were tested and Bruce et.al. taper model was the best fit to the data. Therefore, this model was selected and recommended to predict the volume of individual teak tree for any utilization standard. The volume equation and taper model from this study can confidently applied in the range of the minimum and maximum data (D and H) mentioned above.

Keywords: teak, standard volume equation, taper models

ထုထည်ညီမျှခြင်း နှင့် တေပါ မိုဒယ်များအား နေပြည်တော်၊ မိုးစွေဒေသ၌ တည်ထောင် စိုက်ပျိုးထားသော အနယ်နယ်ကျွန်း သက်ငယ် ကျွန်းစိုက်ခင်းအား အခြေပြုလျက် ဖော်ထုတ်ခြင်း

ဦးရန်မျိုးနိုင်၊ မော်ကွန်းထိန်း၊ သစ်တောနှင့်ပတ်ဝန်းကျင်ဆိုင်ရာ တက္ကသိုလ်၊ရေဆင်း
ဦးဇော်မင်းအေး၊ ဦးစီးအရာရှိ၊ သစ်တောသုတေသနဌာန၊ ရေဆင်း

စာတမ်းအကျဉ်း

ဤစာတမ်းတွင် ထုထည်ညီမျှခြင်းနှင့် တေပါ မိုဒယ်တို့အား နေပြည်တော်၊ မိုးစွေဒေသတွင် တည်ထောင်စိုက်ပျိုးထားသော သက်ငယ်ကျွန်းစိုက်ခင်း (အနယ်နယ်ကျွန်း စမ်းသပ်စိုက်ခင်း) အား ပင်ကျပ်နှုတ်ခုတ်လှဲထားသည့် အပင်များအား တိုင်းတာခြင်းမှ ရရှိလာသော ကိန်းဂဏန်း အချက်အလက်များကို အခြေခံလျက် ဖော်ထုတ်ခဲ့ခြင်း ဖြစ်ပါသည်။ ထုထည် ညီမျှခြင်း (၃) ခုအား ဆန်းစစ်ခြင်းပြုလုပ်ခဲ့ပြီး Furnival's Index(FI) အနည်းဆုံး ညီမျှခြင်းအား သက်ငယ်စိုက်ခင်းများ၏ သစ်ထုထည်အား ခန့်မှန်းနိုင်ရန် ရွေးချယ်ခဲ့ပါသည်။ ရင်စို့ အချင်း (၇) စင်တီ မီတာ မှ (၂၅) စင်တီမီတာ အတွင်း နှင့် အပင် အမြင့် (၈) မီတာ မှ (၂၃) မီတာအတွင်းတွင် ထုထည်အား ယုံကြည်စိတ်ချစွာ တိုင်းတာ နိုင်ပါသည်။ ကျွန်းပင်တစ်ပင် ချင်း၏ ထုထည်မှာ ငုတ်မပါ ထိပ်ဝ အချင်း (၅) စင်တီမီတာ အထိ ဖြစ်ပါသည်။ ဤစာတမ်း၏ ဒုတိယပိုင်းတွင် တေပါ မိုဒယ်အား သက်ငယ်ကျွန်း စိုက်ခင်းအတွက် ဖော်ထုတ်တင်ပြထားပါသည်။ မော်ဒယ် (၃) ခုအား ဆန်းစစ်ခြင်း ပြုလုပ် ခဲ့ရာ Bruce et.al ၏ မော်ဒယ်မှာ သက်ငယ်ကျွန်း စိုက်ခင်းမှ ကောက်ယူရရှိသည့် ကိန်းဂဏန်း အချက်အလက်များနှင့် အကိုက်ညီဆုံး ဖြစ်ကြောင်း တွေ့ရှိရပါသည်။ အဆိုပါ တေပါ မိုဒယ်အား ကျွန်းပင်တစ်ပင်၏ မည်သည့် ဖြတ်ပိုင်းမှဖြစ်စေ ရရှိမည့် သစ်ဖြတ်ပိုင်း ထုထည်အား တွက်ချက်ရယူရာတွင် အသုံးချနိုင်မည်ဖြစ်ပါသည်။ ဤစာတမ်းမှ ရရှိလာသော ထုထည် ညီမျှခြင်းနှင့် တေပါ မိုဒယ်များအား ကောက်ယူထားသည့် အနည်းဆုံးနှင့် အများဆုံး ရင်စို့ အချင်းနှင့် အနည်းဆုံးနှင့် အများဆုံး အမြင့်များအတွင်း ယုံကြည်စိတ်ချစွာ အသုံးပြု နိုင်မည် ဖြစ်ပါသည်။

Development of Volume Equation and Taper Model for Young Teak Plantation: A Case Study from Teak Provenance Trial in Moeswe Area of Nay Pyi Taw

1. Introduction

Myanmar is a tropical country located in Southeast Asia between latitudes 9° 28' and 28° 29' North and longitudes 92° 10' and 101° 10' East. The length from northern to southern tip of the country is about 2,060 kilometers and because of the long stretch of land and the topography, the country has several variant climate zones ranging from the temperate region of the north to the dry zone in central Myanmar and the monsoonal belts in the northwest, west and south (MOF, 2006).

Myanmar is rich in natural resources, particularly forest resources and biodiversity. These valuable resources have been sustainably managed. In fact, scientific and systematic forest management started in Myanmar with the appointment of Dr. Brandis to take charge of the Bago Forests in 1856. In Myanmar, Forest Management Plans (formerly known as working plans) play a key role in managing the forest estates on a sustainable basis. This plan focused on uneven-aged and even-aged forest management.

Nowadays, Forest Department (FD) of Myanmar has established the teak plantations extensively. Moreover, FD has encouraged the private sector to establish the teak plantations since 2006. Additionally, to comply with the international agreements on climate change and restoration of national forest resources, FD have prepared "the Myanmar Reforestation and Rehabilitation Programme (MRRP) (from 2017-2018 to 2026- 2027)" with relevant measures (Natural Regeneration (NR), Assisted NR, Artificial Regeneration (AR) (Enrichment Planting), etc.) rather than the annual forest operational work plan under National Forest Master Plan (NFMP) (2001-2002 to 2030-2031) and are currently being implemented.

However, in Myanmar, there is no standard volume equation and taper model for small dimension of teak trees although it is very important to evaluate the standing timber volume for specified dimension and any utilization standard in order to determine the potential of standing young teak trees in term of volume. Therefore, this study aims to develop standard volume equation and taper model from felled trees.

2. Materials and Methods

2.1. Study Site

The study site is located in compartment no. (9) of Ngalike Reserved Forest under Ottarathiri Forest Township in Moeswe area, Nay Pyi Taw.

2.2. Materials

Materials are the thinning poles of young teak trees from teak provenance trial plantation established in 2007 which were felled during thinning operation in 2021. Minimum diameter at breast height (D) is 7 cm and maximum D is 25 cm. Minimum total height (H) is 8 m and maximum is 23 m.

2.3. Methods

In this study, standard volume equation and taper model are developed.

2.3.1. Development of Standard Volume Equation

Three standard volume equations were tested in this study. Felled trees (158 trees) were available and in each felled tree, diameter at breast height (D) and total height (H) were measured. Stump height was 0.15 m (0.5 ft). Diameter was measured at this point. From D, upper stem diameter was measured at every 1.83 m interval (6 ft) up to approximate 5 cm top diameter. Sectional volume was calculated by Smalian's formula mentioned below. However, the length of the section between 0.15 m and 1.37 m and the last interval length were not 1.83 m intervals. Total volume was the sum of all sectional volumes.

$$V_s = \pi l (d_L^2 + d_U^2)/8$$

Where,

V_s	=	the volume of a section of a stem
l	=	the length of the section
d_L	=	the stem diameter at the lower end of the section (the large end diameter)
d_U	=	the diameter at the upper end of the section (the small end diameter)
π	=	the constant named as pi with the value of 22/7

The three candidate volume models were shown in table 1.

Table 1: List of candidate volume models

No.	Models	References
1.	$V = b_1 + b_2D^2H$	Clutter et al. (1983)
2.	$V = e^{b1}D^{b2}H^{b3}$	Clutter et al. (1983)
	$\ln V = b_1 + b_2 \ln D + b_3 \ln H$	Clutter et al. (1983) Linear form of Model 2
3.	$V = b_1D^2H + b_2D^2H^2$	Bi and Hamilton (1998)

Where,

V	=	volume over bark in cubic meters up to approximate 5 cm top diameter
b_1, b_2, and b_3	=	parameters which will be estimated
D	=	diameter at breast height over bark in centimeters
H	=	total height in meters

The equations included in Table 9 consist of all three categories of double entry volume equations defined by Philip (1994).

- simple combined variable model
- multiple regressions with powers of D and H
- logarithmic forms

To select the best model Furnival's Index (FI) was used. The model with the lowest FI was the best fit to the data.

Furnival's Index (FI)

$$FI = [f'(V)]^{-1}\sqrt{RMSE}$$

Where,

FI	=	Furnival's Index
$f'(V)$	=	the first derivative of dependent variable with respect to V
$RMSE$	=	residual mean square error from fitted regression

$$[f'(V)]^{-1} = [z] = geometric\ mean\ of\ z$$

$$z = \exp\left[\left(\sum_{i=1}^{n} ln\ z_i\right)/n\right]$$

For weighted least square,

Weight variable = D^2H

Weight = $\frac{1}{(D^2H)^k}$

$$FI = \exp\left[\left(p\sum_{i=1}^{n}(ln\ D^2H)_i\right)/n\right]\sqrt{RMSE}$$

Where,

P = a scalar that is half of the value of the power used in weight variable

For logarithmic model,

$$FI = \exp\left[\left(\sum_{i=1}^{n}(ln\ V)_i\right)/n\right]\sqrt{RMSE}$$

All analyzes for volume models were performed in SPSS version 27.

2.3.2. Development of Taper Models

Three taper models were evaluated in this study. They were Bruce et.al (1968) model, Kozak et.al. (1969) model and Demaerschalk-II (1972) model.

2.3.2.1. Bruce et.al. (1968) model

$$\frac{d^2}{D^2} = b_1(X^{1.5})(10^{-1}) + b_2(X^{1.5} - X^3)(D)(10^{-2}) + b_3(X^{1.5} - X^3)(H)(10^{-3}) + b_4(X^{1.5} - X^{32})(D)(H)(10^{-5}) + b_5(X^{1.5} - X^{32})(H^{0.5})(10^{-3}) + b_6(X^{1.5} - X^{40})(H^2)(10^{-6})$$

The ratio of relative diameter (d/D) squared was expressed as a function of D, total height (H) and relative height (h/H). The parameter b_1 in above equation represented squared bark ratio at breast height. The 3/2 and 3rd powers were used to describe the upper four-fifths of the bole, while higher powers (32nd and 40th) were used to describe the butt swell.

$$d = D[b_1(X^{1.5})(10^{-1}) + b_2(X^{1.5} - X^3)(D)(10^{-2}) + b_3(X^{1.5} - X^3)(H)(10^{-3}) + b_4(X^{1.5} - X^{32})(D)(H)(10^{-5}) + b_5(X^{1.5} - X^{32})(H^{0.5})(10^{-3}) + b_6(X^{1.5} - X^{40})(H^2)(10^{-6})]^{0.5}$$

$$X = \frac{H - h}{H - 1.37}$$

To find height **h** above ground at diameter **d**, one must use iterative method; changing the value for **h** in the above equation until the predicted value of **d** is satisfactorily close to the desired value for **d.**

To find sectional volume, use the following formula.

$$V_{LU} = \int_L^U (0.00007854)\, d^2 dh$$

$$V_{LU} = (-0.00007854)(D^2)(-1.37)\left[\frac{2A}{5}(XU^{2.5} - XL^{2.5}) + \frac{B}{4}(XU^4 - XL^4) + \frac{C}{33}(XU^{33} - XL^{33}) + \frac{E}{41}(XU^{41} - XL^{41})\right]$$

$$XU = \frac{H - U}{H - 1.37}$$

$$XL = \frac{H - L}{H - 1.37}$$

$$A = (b_1)(10^{-1}) + b_2(D)(10^{-2}) + (b_3)(H)(10^{-3}) + (b_4)(H)(D)(10^{-5}) + (b_5)(H^{0.5})(10^{-3}) + (b_6)(H^2)(10^{-6})$$

$$B = -[(b_2)(D)(10^{-2}) + (b_3)(H)(10^{-3})]$$

$$C = -[(b_4)(D)(H)(10^{-5}) + (b_5)(H^{0.5})(10^{-3})]$$

$$E = -[(b_6)(H^2)(10^{-6})]$$

Where,

V_{LU} = volume (m^3) over bark between L and U

D = diameter at breast height (cm) (breast height is 1.37 m in this study)

d	=	diameter at height *h* above ground (cm)
h	=	height above ground at diameter *d* (m)
U	=	upper height from ground (m)
L	=	lower height from ground (m)
H	=	total height (m)
b_1 to b_6	=	parameters to be estimated.

2.3.2.2. Kozak et.al. (1969) model

This model was parabolic taper model and normally lack of fit may occur in the lower and upper part of the stem.

$$d^2 = D^2\left(b_1\left(\frac{h}{H}-1\right)+b_2\left(\frac{h^2}{H^2}-1\right)\right)$$

$$h = \frac{(-b_1H)-\sqrt{(b_1H)^2-4b\left(b_0H^2-\frac{d^2H^2}{D^2}\right)}}{2b_2}$$

$$b_0 = -b_1 - b_2$$

$$V_{LU} = \int_L^U (0.00007854)\, d^2 dh$$

$$V_{LU} = [0.00007854\,(D^2)]\left[b_0(U-L)+\frac{b_1(U^2-L^2)}{2H}+\frac{b_2(U^3-L^3)}{3H^2}\right]$$

All the variables are as mentioned above.

2.3.2.3. Demaerschalk-II (1972) model

$$\frac{d^2}{D^2} = (10^{2b_1})(D^{2b_2-2})(H-h)^{2b_3}(H)^{2b_4}$$

$$h = H-[(10^{-b_1})(d)(D^{-b_2})(H^{-b_4})]^{1/b_3}$$

$$V_{LU} = \int_L^U (0.00007854)\, d^2 dh$$

$$V_{LU} = \frac{(0.00007854)(D^2)(10^{2b_1})(D^{2b_2-2})(H^{2b_4})[(H-L)^{2b_3+1}-(H-U)^{2b_3+1}]}{2b_3+1}$$

All the variables are as mentioned above.

2.3.2.4. Taper model evaluation

Models were compared based on four major goodness-of-fit statistics: root mean square error (RMSE), mean difference (MD) between observed and predicted diameter over bark, mean absolute difference (MAD) and R-square (Koirala et.al.,2021). Mean difference and mean absolute difference were also referred as mean bias and mean absolute bias in some literatures e.g. Li and Weiskittel (2010). Lower values of RMSE, MD and MAD and higher value of R-square express better performance of selected equations. In order to better evaluate the models, a ranking technique was employed (Figueiredo-filho et.al., 1996). The rank in ascending orders through rank 3 (the worst value). The model with the lowest total value of ranks was considered as the best model in terms of four goodness-of-fit statistics. All analyzes for taper models were performed in Statistica version 13.5.0.17.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \widehat{Y_i})^2}{n}}$$

$$MD = \frac{\sum_{i=1}^{n}(Y_i - \widehat{Y_i})}{n}$$

$$MAD = \frac{\sum_{i=1}^{n}\left|(Y_i - \widehat{Y_i}\right|}{n}$$

Where, Y_i is the observed diameter over bark; $\widehat{Y_i}$ is the predicted diameter and n is the number of samples.

3. Results and Discussion

3.1 Volume models

The rank of the three models were shown in table 2. Model 1 is the best fitted to the data with the lowest Furnival's Index (FI). Therefore, this model was recommended to calculate the volume of individual young teak tree.

Table 2: Rank of the volume models

Model No.	Power of Weight Variable	P	FI	Rank
1	1.5	7.5	0.0195135	1
2	-	-	0.0205377	3
3	1.5	7.5	0.0197636	2

The fit statistics of model 1 can be seen the table 3. All the parameter were significant at 99% probability level. And also F-value was significant at 99% probability level. Coefficient of determination (R-square) value is high. Standard error is very small.

Table 3 : Fit statistics of volume model 1

Model 1	Value	Significant Level	Standard Error
R^2	0.95	-	-
b_1	0.00504173	0.01	0.001423
b_2	0.00003501	0.01	0.000001
F-Test	-	0.01	-

Figure.1 and 2 showed the predicted volume against observed volume and predicted and observed volumes against variable D^2H.

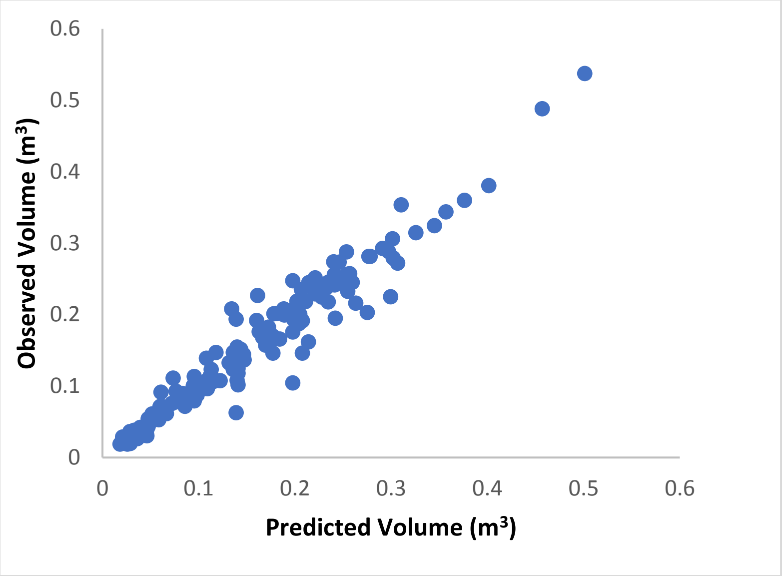

Figure 1. Predicted volume against observed volume

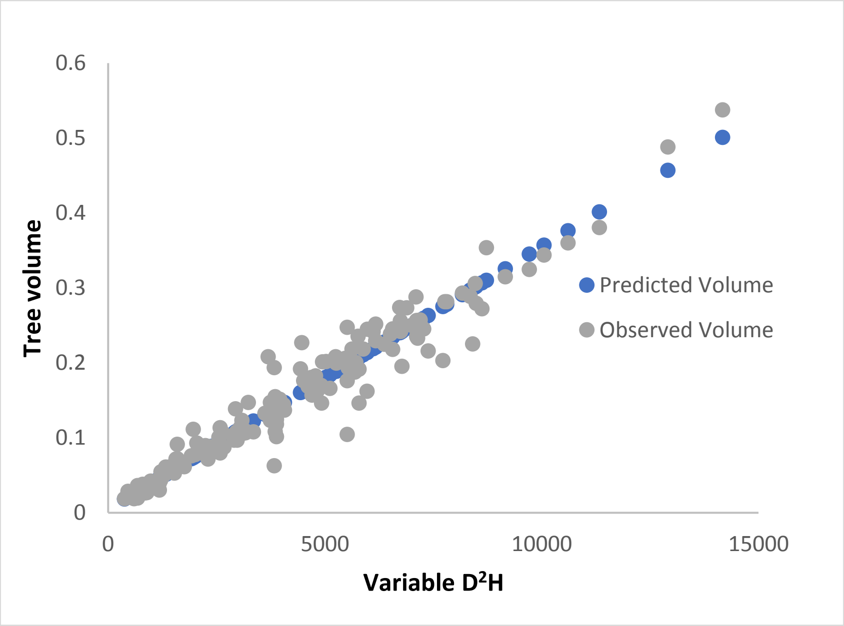

Figure 2. Predicted and observed volumes against D^2H

3.1.1. Model 1 with its parameter values

The following model was the model 1 with its parameter values.

$$V = 0.00504173 + 0.00003501\ D^2H$$

Where,

V = volume of teak tree up to approximate 5 cm top diameter excluding stump (m^3) [stump height is 0.15 m which is equivalent to 0.5 ft]

D = diameter at breast height (cm) [Breast height is 1.37 m from the ground equivalent to 4.5 ft]

H = total height (m)

3.2. Taper models

The fit statistics of three taper models were given in table 4.

Table 4: Fit statistics of taper models

Model	b_1	b_2	b_3	b_4	b_5	b_6	F-Test
Bruce	10.821**	3.263**	-46.344**	15.853**	18.237**	154.483**	**
Dema	0.24327**	0.86973**	1.23511**	1.22898**	-	-	**
Kozak	3.01391**	1.53528**	-	-	-	-	**

** means significant at 99% probability level.

The overall rank for each taper model was shown in table 5. In the table, Bruce et.al. model (model 1) is the lowest rank and it was recommended to estimate the volume of teak tree of any utilization standard.

Table 5: Ranks for taper models

Model	Name	R-squared	Rank	RMSE	Rank	MD	Rank	MAD	Rank	Overall
1	Bruce	0.884773	1	1.623298	2	0.150929	1	1.182379	1	5
2	Dema	0.776692	2	2.65466	3	1.005655	3	2.162918	3	11
3	Kozak	0.758937	3	1.198103	1	0.411305	2	1.877827	2	8

Figure 3 showed the observed diameter over bark against predicted diameter over bark.

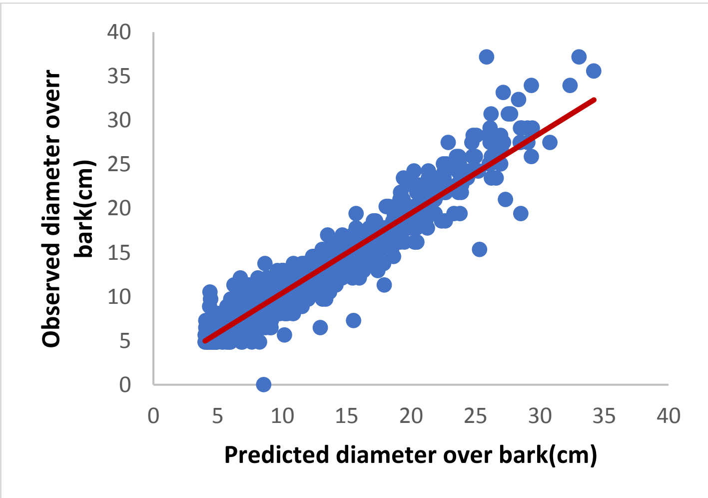

Figure 3. Observed diameter over bark against predicted diameter over bark

Figure 4 showed that while relative height (h/H) approaches to 1, relative diameter (d/D) approaches to 0.

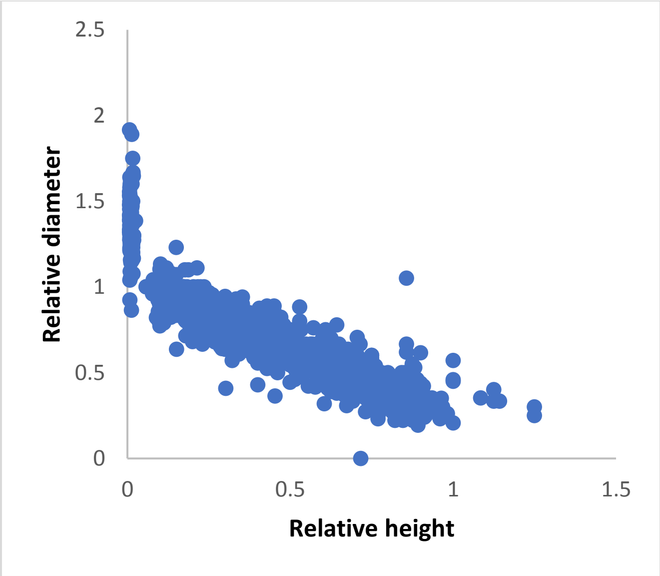

Figure 4. Scatter plot of relative diameter (d/D) over relative height(h/H)

Actual and predicted height and diameter over bark can be seen in Figure 5 for the highest tree, in Figure 6 for medium height tree and in Figure 7 for the shortest trees respectively.

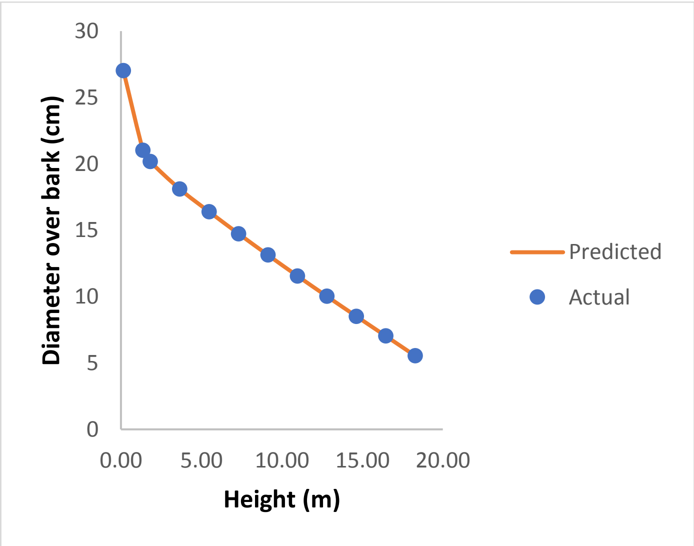

Figure 5. Height and diameter relationship of the highest tree

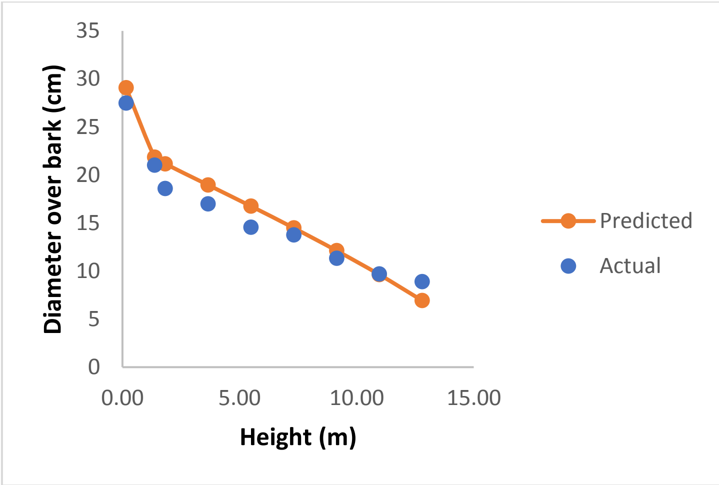

Figure 6. Height and diameter relationship of the medium height tree

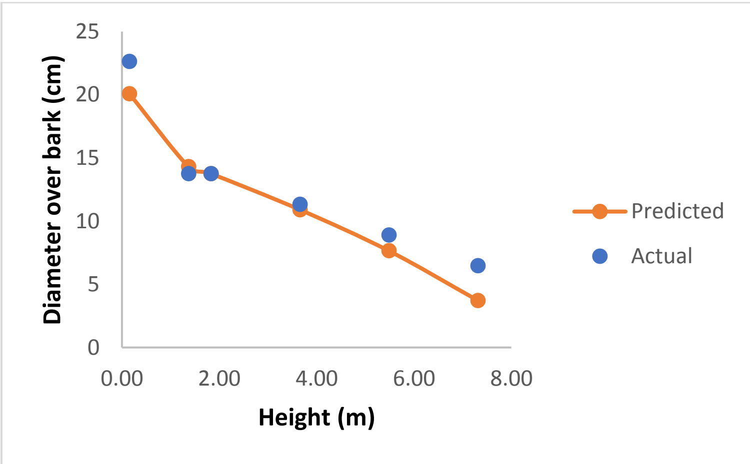

Figure 7. Height and diameter relationship of the shortest tree

4. Comparisons of actual volume with predicted volumes from volume and taper models

The table 6 showed comparisons among the actual volume and volumes from volume and taper models. All the calculated volumes with the same D and H showed very similar amount and not so much different. (0.01 m^3 different from 1 and 2 i.e. only 1 cm^3).

Table 6: Comparisons of volumes from actual calculation and different models

	Actual Volume Calculated (1)	Volume from Volume Model (1) (2)	Volume from Taper Model (1) (3)
D 15.36 H 14.20	0.11	0.11	0.12

(1) and (2) are volumes (m^3) up to approximate 5cm top diameter excluding stump.
(3) The sectional volume (m^3) between 0.15 m height (stump height) and 11.5 m height (approximate 5 cm top diameter)

5. Conclusion

The present study aimed to develop standard volume equation and taper model for small dimension teak tree. The data came from felled trees during thinning operation of teak provenance trial plantation established in compartment no. (9) of Ngalike Reserved Forest under Ottarathiri Forest Township in Moeswe area, Nay Pyi Taw. The three volume equations and three taper models were analyzed. To select the best volume equation, FI was used. The equation with the lowest FI was model 1. This standard volume equation was recommended to calculate the standing teak tree volume within the minimum and maximum data range. Three taper models were evaluated by RMSE, MD, MAD and R- square values. The model with lowest overall rank was Bruce et.al. (1968) taper model and this model was recommended to calculate the volume for any utilization standard within the minimum and maximum data range. Finally, it was believed that this study will help the user effectively in determining the volume of standing young teak tree for specified top diameter (5 cm) excluding stump by applying two entry volume equation and the volume of the standing tree for any utilization standard by applying taper model.

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